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ABSTRACT

This paper examines the often stated idea that the poor do not support high levels of redistribution because of the hope that they, or their offspring, may make it up the income ladder. This “prospect of upward mobility” (POUM) hypothesis is shown to be fully compatible with rational expectations, and fundamentally linked to concavity in the mobility process. A steady-state majority could even be simultaneously poorer than average in terms of current income, and richer than average in terms of expected future incomes. A first empirical assessment suggests, on the other hand, that in recent U. S. data the POUM effect is probably dominated by the demand for social insurance.

“In the future, everyone will be world-famous for fifteen minutes”

[Andy Warhol 1968].

INTRODUCTION

The following argument is among those commonly advanced to explain why democracies, where a relatively poor majority holds the political power, do not engage in large-scale expropriation and redistribution. Even people with income below average, it is said, will not support high tax rates because of the prospect of upward mobility: they take into account the fact that they, or their children, may move up in the income distribution and therefore be hurt by such policies.(FN1) For instance, Okun [1975, p. 49] relates that: “In 1972 a storm of protest from blue-collar workers greeted Senator McGovern’s proposal for confiscatory estate taxes. They apparently wanted some big prizes maintained in the game. The silent majority did not want the yacht clubs closed forever to their children and grandchildren while those who had already become members kept sailing along.”

The question we ask in this paper is simple: does this story make sense with economic agents who hold rational expectations over their income dynamics, or does it require that the poor systematically overestimate their chances of upward mobility—a form of what Marxist writers refer to as “false consciousness”?(FN2)

The “prospect of upward mobility” (POUM) hypothesis has, to the best of our knowledge, never been formalized, which is rather surprising for such a recurrent theme in the political economy of redistribution. There are three implicit premises behind this story. The first is that policies chosen today will, to some extent, persist into future periods. Some degree of inertia or commitment power in the setting of fiscal policy seems quite reasonable. The second assumption is that agents are not too risk averse, for otherwise they must realize that redistribution provides valuable insurance against the fact that their income may go down as well as up. The third and key premise is that individuals or families who are currently poorer than average—for instance, the median voter—expect to become richer than average. This “optimistic” view clearly cannot be true for everyone below the mean, barring the implausible case of negative serial correlation. Moreover, a standard mean-reverting income process would seem to imply that tomorrow’s expected income lies somewhere between today’s income and the mean. This would leave the poor of today still poor in relative terms tomorrow, and therefore demanders of redistribution. And even if a positive fraction of

agents below the mean today can somehow expect to be above it tomorrow, the expected incomes of those who are currently richer than they must be even higher. Does this not then require that the number of people above the mean be forever rising over time, which cannot happen in steady state? It thus appears—and economists have often concluded—that the intuition behind the POUM hypothesis is flawed, or at least incompatible with everyone holding realistic views of their income prospects.(FN3)

The contribution of this paper is to formally examine the prospect of upward mobility hypothesis, asking whether and when it can be valid. The answer turns out to be surprisingly simple, yet a bit subtle. We show that there exists a range of incomes below the mean where agents oppose lasting redistributions if (and, in a sense, only if) tomorrow's expected income is an increasing and concave function of today's income. The more concave the transition function, and the longer the length of time for which taxes are preset, the lower the demand for redistribution. Even the median voter—in fact, even an arbitrarily poor voter—may oppose redistribution if either of these factors is large enough. We also explain how the concavity of the expected transition function and the skewness of idiosyncratic income shocks interact to shape the long-run distribution of income. We construct, for instance, a simple Markov process whose steady-state distribution has three-quarters of the population below mean income, so that they would support purely contemporary redistributions. Yet when voters look ahead to the next period, two-thirds of them have expected incomes above the mean, and this super-majority will therefore oppose (perhaps through constitutional design) any redistributive policy that bears primarily on future incomes.

Concavity of the expected transition function is a rather natural property, being simply a form of decreasing returns: as current income rises, the odds for future income improve, but at a decreasing rate. While this requirement is stronger than simple mean reversion or convergence of individual incomes, concave transition functions are ubiquitous in economic models and econometric specifications. They arise, for instance, when current resources affect investment due to credit constraints and the accumulation technology has decreasing returns; or when some income-generating individual characteristic, such as ability, is passed on to children according to a similar "technology." In particular, the specification of income dynamics most widely used in theoretical and empirical work, namely the log linear $ar(1)$ process, has this property.

The key role played by concavity in the POUM mechanism may be best understood by first considering a very stripped-down example. Suppose that agents decide today between "laissez-faire" and complete sharing with respect to next period's income, and that the latter is a deterministic function of current income: $y' = f(y)$, for all y in some interval $[0, \bar{y}]$. Without loss of generality, normalize f so that someone with income equal to the average, μ , maintains that same level tomorrow ($f(\mu) = \mu$). As shown in Figure 1, everyone who is initially poorer will then see his income rise, and conversely all those who are initially richer will experience a decline. The concavity of f —more specifically, Jensen's inequality—means that the losses of the rich sum to more than the gains of the poor; therefore, tomorrow's per capita income μ' is below μ . An agent with mean initial income, or even somewhat poorer, can thus rationally expect to be richer than average in the next period, and will therefore oppose future redistributions.

To provide an alternative interpretation, let us now normalize the transition function so that tomorrow's and today's mean incomes coincide: $\mu' = \mu$. The concavity of f can then be interpreted as saying that y' is obtained from y through a progressive, balanced budget, redistributive scheme, which shifts the Lorenz curve upward and reduces the skewness of the income distribution. As is well-known, such progressivity leaves the individual with average endowment better off than under laissez-faire,

because income is taken disproportionately from the rich. This means that the expected income y' of a person with initial income μ is strictly greater than μ , hence greater than the average of y' across agents. This person, and those with initial incomes not too far below, will therefore be hurt if future incomes are redistributed.(FN4)

Extending the model to a more realistic stochastic setting brings to light another important element of the story, namely the skewness of idiosyncratic income shocks. The notion that life resembles a lottery where a lucky few will “make it big” is somewhat implicit in casual descriptions of the POUM hypothesis—such as Okun’s. But, in contrast to concavity, skewness in itself does nothing to reduce the demand for redistribution; in particular, it clearly does not affect the distribution of expected incomes. The real role played by such idiosyncratic shocks, as we show, is to offset the skewness-reducing effect of concave expected transitions functions, so as to maintain a positively skewed distribution of income realizations (especially in steady state). The balance between the two forces of concavity and skewness is what allows us to rationalize the apparent risk-loving behavior, or overoptimism, of poor voters who consistently vote for low tax rates due to the slim prospects of upward mobility.

With the important exceptions of Hirschman [1973] and Piketty [1995a, 1995b], the economic literature on the implications of social mobility for political equilibrium and redistributive policies is very sparse. For instance, mobility concerns are completely absent from the many papers devoted to the links between income inequality, redistributive politics, and growth (e.g., Alesina and Rodrik [1994] and Persson and Tabellini [1994]). A key mechanism in this class of models is that of a poor median voter who chooses high tax rates or other forms of expropriation, which in turn discourage accumulation and growth. We show that when agents vote not just on the current fiscal policy but on one that will remain in effect for some time, even a poor median voter may choose a low tax rate—independently of any deadweight loss considerations.

While sharing the same general motivation as Piketty [1995a, 1995b], our approach is quite different. Piketty’s main concern is to explain persistent differences in attitudes toward redistribution. He therefore studies the inference problem of agents who care about a common social welfare function, but learn about the determinants of economic success only through personal or dynastic experimentation. Because this involves costly effort, they may end up with different long-run beliefs over the incentive costs of taxation. We focus instead on agents who know the true (stochastic) mobility process and whose main concern is to maximize the present value of their aftertax incomes, or that of their progeny. The key determinant of their vote is therefore how they assess their prospects for upward and downward mobility, relative to the rest of the population.

The paper will formalize the intuitions presented above linking these relative income prospects to the concavity of the mobility process and then examine their robustness to aggregate uncertainty, longer horizons, discounting, risk aversion, and nonlinear taxation. It will also present an analytical example that demonstrates how a large majority of the population can be simultaneously below average in terms of current income and above average in terms of expected future income, even though the income distribution remains invariant.(FN5) Interestingly, a simulated version of this simple model fits some of the main features of the U. S. income distribution and intergenerational persistence rather well. It also suggests, on the other hand, that the POUM effect can have a significant impact on the political equilibrium only if agents have relatively low degrees of risk aversion.

Finally, the paper also makes a first pass at the empirical assessment of the POUM hypothesis. Using interdecile mobility matrices from the PSID, we compute over

different horizons the proportion of agents who have expected future incomes above the mean. Consistent with the theory, we find that this laissez-faire coalition grows with the length of the forecast period, to reach a majority for a horizon of about twenty years. We also find, however, that these expected income gains of the middle class are likely to be dominated, under standard values of risk aversion, by the desire for social insurance against the risks of downward mobility or stagnation.

I. PRELIMINARIES

We consider an endowment economy populated by a continuum of individuals indexed by $i \in [0,1]$, whose initial levels of income lie in some interval $X \equiv [0, \bar{y}]$, $0 < \bar{y} \leq \infty$.(FN6) An income distribution is defined as a continuous function $F: X \rightarrow [0,1]$ such that $F(0) = 0$, $F(\bar{y}) = 1$ and $\mu_F \equiv \int_X y dF < \infty$. We shall denote by F the class of all such distributions, and by F_+ the subset of those whose median, $m_F \equiv \min [F^{-1}(\frac{1}{2})]$, is below their mean. We shall refer to such distributions as positively skewed, and more generally we shall measure "skewness" in a random variable as the proportion of realizations below the mean (minus a half), rather than by the usual normalized third moment.

A redistribution scheme is defined as a function $r: X \times F \rightarrow R_+$ which assigns to each pretax income and initial distribution a level of disposable income $r(y; F)$, while preserving total income: $\int_X r(y; F) dF(y) = \mu_F$. We thus abstract from any deadweight losses that such a scheme might realistically entail, in order to better highlight the different mechanism which is our focus. Both represent complementary forces reducing the demand for redistribution, and could potentially be combined into a common framework.

The class of redistributive schemes used in a vast majority of political economy models is that of proportional schemes, where all incomes are taxed at the rate τ and the collected revenue is redistributed in a lump-sum manner.(FN7) We denote this class as $P \equiv [r_\tau | 0 \leq \tau \leq 1]$, where $r_\tau(y; F) \equiv (1 - \tau)y + \tau\mu_F$ for all $y \in X$ and $F \in F$. We shall mostly work with just the two extreme members of P , namely, r_0 and r_1 . Clearly, r_0 corresponds to the "laissez-faire" policy, whereas r_1 corresponds to "complete equalization."

Our focus on these two polar cases is not nearly so restrictive as might initially appear. First, the analysis immediately extends to the comparison between any two proportional redistribution schemes, say r_τ and $r_{\tau'}$, with $0 \leq \tau < \tau' \leq 1$. Second, r_0 and r_1 are in a certain sense "focal" members of P since, in the simplest framework where one abstracts from taxes' distortionary effects as well as their insurance value, these are the only candidates in this class that can be Condorcet winners. In particular, for any distribution with median income below the mean, r_1 beats every other linear scheme under majority voting if agents care only about their current disposable income. We shall see that this conclusion may be dramatically altered when individuals' voting behavior also incorporates concerns about their future incomes. Finally, in Section IV we shall extend the analysis to nonlinear (progressive or regressive) schemes, and show that our main results remain valid.

As pointed out earlier, mobility considerations can enter into voter preferences only if current policy has lasting effects. Such persistence is quite plausible given the many sources of inertia and status quo bias that characterize the policy-making process, especially in an uncertain environment. These include constitutional limits on the frequency of tax changes, the costs of forming new coalitions and passing new legislation, the potential for prolonged gridlock, and the advantage of incumbent candidates and parties in electoral competitions. We shall therefore take such persistence as given, and formalize it by assuming that tax policy must be set one

period in advance, or more generally present for T periods. We will then study how the length of this commitment period affects the demand for redistribution.(FN8)

The third and key feature of the economy is the mobility process. We shall study economies where individual incomes or endowments y_t^i evolve according to a law of motion of the form,

$$(1) y_{t+1}^i = f(y_t^i, \theta_{t+1}^i), \quad t = 0, \dots, T - 1,$$

where f is a stochastic transition function and θ_{t+1}^i is the realization of a random shock Θ_{t+1}^i .(FN9) We require this stochastic process to have the following properties:

(i) The random variables $\Theta_{i,t}^i \in [0, 1] \times [1, \dots, T]$, have a common probability distribution function P , with support Ω .

(ii) The function $f : X \times \Omega \rightarrow X$ is continuous, with a well-defined expectation $E_\theta[f(\cdot; \Theta)]$ on X .

(iii) Future income increases with current income, in the sense of first-order stochastic dominance: for any $(y, y') \in X^2$, the conditional distribution $M(y'|y) \equiv \text{prob}(\{\theta \in \Omega | f(y; \theta) \leq y'\})$ is decreasing in y , with strict monotonicity on some nonempty interval in X .

The first condition means that everyone faces the same uncertain environment, which is stationary across periods. Put differently, current income is the only individual-level state variable that helps predict future income. While this focus on unidimensional processes follows a long tradition in the study of socioeconomic mobility (e.g., Atkinson [1983], Shorrocks [1978], Conlisk [1990], and Dardanoni [1993]), one should be aware that it is fairly restrictive, especially in an intragenerational context. It means, for instance, that one abstracts from life-cycle earnings profiles and other sources of lasting heterogeneity such as gender, race, or occupation, which would introduce additional state variables into the income dynamics. This becomes less of a concern when dealing with intergenerational mobility, where one can essentially think of the two-period case, $T = 1$, as representing overlapping generations. Note, finally, that condition (i) puts no restriction on the correlation of shocks across individuals; it allows for purely aggregate shocks ($\Theta_i^i = \Theta_j^j$ for all i, j in $[0, 1]$), purely idiosyncratic shocks (the Θ_i^i 's are independent across agents and sum to zero), and all cases in between.(FN10)

The second condition is a minor technical requirement. The third condition implies that expected income $E_\theta[f(\cdot; \Theta)]$ rises with current income, which is what we shall actually use in the results. We impose the stronger distributional monotonicity for realism, as all empirical studies of mobility (intra- or intergenerational) find income to be positively serially correlated and transition matrices to be monotone. Thus, given the admittedly restrictive assumption (i), (iii) is a natural requirement to impose.

We shall initially focus the analysis on deterministic income dynamics (where θ_t^i is just a constant, and therefore dropped from the notation), and then incorporate random shocks. While the stochastic case is obviously of primary interest, the deterministic one makes the key intuitions more transparent, and provides useful intermediate results. This two-step approach will also help highlight the fundamental dichotomy between the roles of concavity in expectations and skewness in realizations.

II. INCOME DYNAMICS AND VOTING UNDER CERTAINTY

It is thus assumed for now that individual pretax incomes or endowments evolve according to a deterministic transition function f , which is continuous and strictly increasing. The income stream of an individual with initial endowment $y \in X$ is then $y, f(y), f^2(y), \dots, f^t(y), \dots$, and for any initial $F \in F$ the cross-sectional distribution of incomes in period t is $F_t \equiv F$ [Graphic Character Omitted] f^t . A particularly interesting class of transition functions for the purposes of this paper is the set of all concave (but not affine transition functions; we denote this set by T .

II.A. TWO-PERIOD ANALYSIS

To distill our main argument into its most elementary form, we focus first on a two-period (or overlapping generations) scenario, where individuals vote “today” (date 0) over alternative redistribution schemes that will be enacted only “tomorrow” (date 1). For instance, the predominant motive behind agents’ voting behavior could be the well-being of their offspring, who will be subject to the tax policy designed by the current generation. Accordingly, agent $y \leq X$ votes for r_1 over r_0 if she expects her period 1 earnings to be below the per capita average:

$$(2) \quad f(y) < \int_X f dF_0 = \mu_{F_1}$$

Suppose now that $f \leq T$; that is, it is concave but not affine. Then, by Jensen’s inequality,

$$(3) \quad f(\mu_{F_0}) = f(\int_X y dF_0) > \int_X f dF_0 = \mu_{F_1},$$

so the agent with average income at date 0 will oppose date 1 redistributions. On the other hand, it is clear that $f(0) < \mu_{F_1}$, so there must exist a unique y^*_f in $(0, \mu_{F_0})$ such that $f(y^*_f) \equiv \mu_{F_1}$. Of course $y^*_f = f^{-1}(\mu_{F_1})$ also depends on F_0 but, for brevity, we do not make this dependence explicit in the notation. Since f is strictly increasing, it is clear that y^*_f acts as a tipping point in agents’ attitudes toward redistributions bearing on future income. Moreover, since Jensen’s inequality—with respect to all distributions F_0 —characterizes concavity, the latter is both necessary and sufficient for the prospect of upward mobility hypothesis to be valid, under any linear redistribution scheme.(FN11)

PROPOSITION 1. The following two properties of a transition function f are equivalent:

(a) f is concave (but not affine); i.e., $f \leq T$.

(b) For any income distribution $F_0 \leq F$ there exists a unique $y^*_f < \mu_{F_0}$ such that all agents in $[0, y^*_f)$ vote for r_1 over r_0 , while all those in $(y^*_f, \bar{y}]$ vote for r_0 over r_1 .

Yet another way of stating the result is that $f \leq T$ if it is skewness-reducing: for any initial F_0 , next period’s distribution $F_1 = F_0$ [Graphic Character Omitted] f^{-1} is such that $F_1(\mu_{F_1}) < F_0(\mu_{F_0})$. Compared with the standard case where individuals base their votes solely on how taxation affects their current disposable income, popular support for redistribution thus falls by a measure $F_0(\mu_{F_0}) - F_1(\mu_{F_1}) = F_0(\mu_{F_0}) - F_0(y^*_f) > 0$. The underlying intuition also suggests that the more concave is the transition function, the fewer people should vote for redistribution. This simple result will turn out to be very useful in establishing some of our main propositions on the outcome of majority voting and on the effect of longer political horizons.

We shall say that $f \leq T$ is more concave than $g \leq T$, and write f [Graphic Character Omitted] g , if and only if f is obtained from g through an increasing and concave (not affine) transformation; that is, if there exists an $h \leq T$ such that $f = h$ [Graphic Character Omitted] g . Put differently, f [Graphic Character Omitted] g if and only if f [Graphic Character Omitted] $g^{-1} \leq T$.

PROPOSITION 2. Let $F_0 \leq F$ and $f, g \leq T$. Then f [Graphic Character Omitted] g implies that $y^*_f < y^*_g$.

The underlying intuition is, again, straightforward: the demand for future fiscal redistribution is lower under the transition process which reduces skewness by more. Can prospects of upward mobility be favorable enough for r_0 to beat r_1 under majority voting? Clearly, the outcome of the election depends on the particular characteristics of f and F_0 . One can show, however, that for any given pretax income distribution F_0 there exists a transition function f which is “concave enough” that a majority of voters choose laissez-faire over redistribution.(FN12) When combined with Proposition 2, it allows us to show the following, more general result.

THEOREM 1. For any $F_0 \leq F_+$, there exists an $f \leq T$ such that r_0 beats r_1 under pairwise majority voting for all transition functions that are more concave than f , and r_1 beats r_0 for all transition functions that are less concave than f .

This result is subject to an obvious caveat, however: for a majority of individuals to vote for laissez-faire at date 0, the transition function must be sufficiently concave to make the date 1 income distribution F_1 negatively skewed. Indeed, if $y_f^* = f^{-1}(\mu_{F_1}) < m_{F_0}$, then $\mu_{F_1} < f(m_{F_0}) = m_{F_1}$. There are two reasons why this is far less problematic than might initially appear. First and foremost, it simply reflects the fact that we are momentarily abstracting from idiosyncratic shocks, which typically contribute to reestablishing positive skewness. Section III will thus present a stochastic version of Theorem 1, where F_1 can remain as skewed as one desires. Second, it may in fact not be necessary that the cutoff y_f^* fall all the way below the median for redistribution to be defeated. Even in the most developed democracies it is empirically well documented that poor individuals have lower propensities to vote, contribute to political campaigns, and otherwise participate in the political process, than rich ones. The general message of our results can then be stated as follows: the more concave the transition function, the smaller the departure from the "one person, one vote" ideal needs to be for redistributive policies, or parties advocating them, to be defeated.

II.B. MULTIPERIOD REDISTRIBUTIONS

In this subsection we examine how the length of the horizon over which taxes are set and mobility prospects evaluated affects the political support for redistribution. We thus make the more realistic assumption that the tax scheme chosen at date 0 will remain in effect during periods $t = 0, \dots, T$, and that agents care about the present value of their disposable income stream over this entire horizon. Given a transition function f and a discount factor $\delta \in (0, 1]$, agent $y \in X$ votes for laissez-faire over complete equalization if

(4) $[y > \delta \sum_{t=0}^T f^t(y)]$ where we recall that f^t denotes the t th iteration of f and $F_t \equiv F_0 \circ f^t$ is the period t income distribution, with mean μ_{F_t} .

We shall see that there again exists a unique tipping point $y_f^*(T)$ such that all agents with initial income less than $y_f^*(T)$ vote for r_1 , while all those richer than $y_f^*(T)$ vote for r_0 . When the policy has no lasting effects, this point coincides with the mean: $y_f^*(0) = \mu_{F_0}$. When future incomes are factored in, the coalition in favor of laissez-faire expands: $y_f^*(T) < \mu_{F_0}$ for $T \geq 1$. In fact, the more farsighted voters are, or the longer the duration of the proposed tax scheme, the less support for redistribution there will be: $y_f^*(T)$ is strictly decreasing in T . If agents care enough about future incomes, the increase in the vote for r_0 can be enough to ensure its victory over r_1 .

THEOREM 2. Let $F \leq F_+$ and $\delta \in (0, 1]$.

(a) For all $f \leq T$, the longer is the horizon T , the larger is the share of votes that go to r_0 .

(b) For all δ and T large enough, there exists an $f \leq T$ such that r_0 ties with r_1 under pairwise majority voting. Moreover, r_0 beats r_1 if the duration of the redistribution scheme is extended beyond T , and is beaten by r_1 if this duration is reduced below T .

Simply put, longer horizons magnify the prospect of upward mobility effect, whereas discounting works in the opposite direction. The intuition is very simple, and related to Proposition 2: when forecasting incomes further into the future, the one-step transition f gets compounded into f^2, \dots, f^T , etc., and each of these functions is more concave than its predecessor. (FN13)

III. INCOME DYNAMICS AND VOTING UNDER UNCERTAINTY

The assumption that individuals know their future incomes with certainty is obviously unrealistic. Moreover, in the absence of idiosyncratic shocks the cross-sectional distribution becomes more equal over time, and eventually converges to a single masspoint. In this section we therefore extend the analysis to the stochastic case, while maintaining risk neutrality. The role of insurance will be considered later on.

Income dynamics are now governed by a stochastic process $y_{i,t+1} = f(y_i; \theta_{i,t+1})$ satisfying the basic requirements (i)-(iii) discussed in Section I, namely stationarity, continuity, and monotonicity. In the deterministic case the validity of the POUM conjecture was seen to hinge upon the concavity of the transition function. The strictest extension of this property to the stochastic case is that it should hold with probability one. Therefore, let T_P be the set of transition functions such that $\text{prob} [\Theta f(\cdot; \theta) \leq T] = 1$. It is clear that, for any f in T_P ,

(iv) The expectation $E\Theta[f(\cdot; \Theta)]$ is concave (but not affine) on X .

For some of our purposes the requirement that $f \in T_P$ will be too strong, so we shall develop our analysis for the larger set of mobility processes that simply satisfy concavity in expectation. We shall therefore denote as T_P^* the set of transition functions that satisfy conditions (i) to (iv).

III.A. TWO-PERIOD ANALYSIS

We first return to the basic case where risk-neutral agents vote in period 0 over redistributing period 1 incomes. Agent $y^i \in X$ then prefers r_0 to r_1 if and only if

$$(5) E_{\Theta^i}[f(y^i; \Theta^i)] > E[\mu_{F_1}],$$

where the subscript Θ^i on the left-hand side indicates that the expectation is taken only with respect to Θ^i , for given y^i . When shocks are purely idiosyncratic, the future mean μ_{F_1} is deterministic due to the law of large numbers; with aggregate shocks it remains random. In any case, the expected mean income at date 1 is the mean expected income across individuals:

$$E[\mu_{F_1}] = E[\int_0^1 f(y^j; \Theta^j) dj] = \int_0^1 E[f(y^j; \Theta^j)] dj = \int_X E_{\Theta}[f(y; \Theta)] dF_0(y),$$

by Fubini's theorem. This is less than the expected income of an agent whose initial endowment is equal to the mean level μ_{F_0} , whenever $f(y; \theta)$ —or, more generally, $E_{\Theta}[f(y; \Theta)]$ —is concave in y :

$$(6) \int_X E_{\Theta}[f(y; \Theta)] dF_0(y) < E_{\Theta}[f(\mu_{F_0}; \Theta)].$$

Consequently, there must again exist a nonempty interval of incomes $[y_f^*, \mu_{F_0}]$ in which agents will oppose redistribution, with the cutoff y_f^* defined by

$$E_{\Theta}[f(y_f^*; \Theta)] = E[\mu_{F_1}].$$

The basic POUM result thus holds for risk-neutral agents whose incomes evolve stochastically. To examine whether an appropriate form of concavity still affects the cutoff monotonically, and whether enough of it can still cause r_0 to beat r_1 under majority voting, observe that the inequality in (6) involves only the expected transition function $E_{\Theta}[f(y; \Theta)]$, rather than f itself. This leads us to replace the “more concave than” relation with a “more concave in expectation than” relation. Given any probability distribution P , we define this ordering on the class T_P^* as

$$f >_P g \text{ if and only if } E_{\Theta}[f(\cdot; \Theta)] > E_{\Theta}[g(\cdot; \Theta)],$$

where Θ is any random variable with distribution P . (FN14) It is easily shown that $f >_P g$ implies that $y_f^* < y_g^*$. In fact, making f concave enough in expectation will, as before, drive the cutoff y_f^* below the median m_{F_0} , or even below any chosen income level. Most importantly, since this condition bears only on the mean of the random function $f(\cdot; \Theta)$, it puts essentially no restriction on the skewness of the period 1 income distribution F_1 —in sharp contrast to what occurred in the deterministic case. In particular, a sufficiently skewed distribution of shocks will ensure that $F_1 \leq F_+$ without

affecting the cutoff y_f^* . This dichotomy between expectations and realizations is the second key component of the POUM mechanism, and allows us to establish a stochastic generalization of Theorem 1.(FN15)

THEOREM 3. For any $F_0 \leq F_+$ and any $\sigma \leq (0, 1)$, there exists a mobility process (f, P) with $f \leq T_P^*$ such that $F_1(\mu_{F_1}) \geq \sigma$ and, under pairwise majority voting, r_0 beats r_1 for all transition functions in T_P^* that are more concave than f in expectation, while r_1 beats r_0 for all those that are less concave than f in expectation.

Thus, once random shocks are incorporated, we reach essentially the same conclusions as in Section II, but with much greater realism. Concavity of $E_\theta[f(\cdot; \Theta)]$ is necessary and sufficient for the political support behind the laissez-faire policy to increase when individuals' voting behavior takes into account their future income prospects. If f is concave enough in expectation, then r_0 can even be the preferred policy of a majority of voters.

III.B. STEADY-STATE DISTRIBUTIONS

The presence of idiosyncratic uncertainty is not only realistic, but is also required to ensure a nondegenerate long-run income distribution. This, in turn, is essential to show that our previous findings describe not just transitory, short-run effects, but stable, permanent ones as well.

Let P be a probability distribution of idiosyncratic shocks and f a transition function in T_P^* . An invariant or steady-state distribution of this stochastic process is an $F \leq F$ (not necessarily positively skewed) such that

$$F(y) = \int_{GQ} \int_X 1_{[f(x, \theta) \leq y]} dF(x) dP(\theta) \text{ for all } y \leq X,$$

where $1_{[\cdot]}$ denotes the indicator function. Since the basic result that the coalition opposed to lasting redistributions includes agents poorer than the mean holds for all distributions in F , it applies to invariant ones in particular: thus, $y_f^* < \mu_F$.(FN16)

This brings us back to the puzzle mentioned in the introduction. How can there be a stationary distribution F where a positive fraction of agents below the mean μ_F have expected incomes greater than μ_F , as do all those who start above this mean, given that the number of people on either side of μ_F must remain invariant over time?

The answer is that even though everyone makes unbiased forecasts, the number of agents with expected income above the mean, $1 - F(y_f^*, F)$, strictly exceeds the number who actually end up with realized incomes above the mean, $1 - F(\mu_F)$, whenever f is concave in expectation. This result is apparent in Figure II, which provides additional intuition by plotting each agent's expected income path over future dates, $E[y^i | y^i]$. In the long run everyone's expected income converges to the population mean μ_F , but this convergence is nonmonotonic for all initial endowments in some interval (y_F, \bar{y}_F) around μ_F . In particular, for $y^i_0 \leq (y_F, \mu_F)$ expected income first crosses the mean from below, then converges back to it from above. While such nonmonotonicity may seem surprising at first, it follows from our results that all concave (expected) transition functions must have this feature.

This still leaves us with one of the most interesting questions: can one find income processes whose stationary distribution is positively skewed, but where a strict majority of the population nonetheless opposes redistribution? The answer is affirmative, as we shall demonstrate through a simple Markovian example. Let income take one of three values: $X = [a_1, a_2, a_3]$, with $a_1 < a_2 < a_3$. The transition probabilities between these states are independent across agents, and given by the Markov matrix:

$$(7) M = [\text{Formula Omitted}]$$

where $(p, q, r, s) \leq (0, 1)_4$. The invariant distribution induced by M over $[a_1, a_2, a_3]$ is found by solving $\pi M = \pi$. It will be denoted by $\pi = (\pi_1, \pi_2, \pi_3)$, with mean $\mu \equiv \pi_1 a_1 + \pi_2 a_2 + (1 - \pi_1 - \pi_2) a_3$. We require that the mobility process and associated steady state satisfy the following conditions:

- (a) Next period's income $y_{i,t+1}^i$ is stochastically increasing in current income y_t^i ; (FN17)
 (b) The median income level is a_2 : $\pi_1 < \frac{1}{2} < \pi_1 + \pi_2$;
 (c) The median agent is poorer than the mean: $a_2 < \mu$;
 (d) The median agent has expected income above the mean: $E[y_{i,t+1}^i | y_t^i = a_2] > \mu$.

Conditions (b) and (c) together ensure that a strict majority of the population would vote for current redistribution, while (b) and (d) together imply that a strict majority will vote against future redistribution. In Bénabou and Ok [1998] we provide sufficient conditions on $(p, q, r, s; a_1, a_2, a_3)$ for (a)-(d) to be satisfied, and show them to hold for a wide set of parameters. In the steady state of such an economy the distribution of expected incomes is negatively skewed, even though the distribution of actual incomes remains positively skewed and every one has rational expectations.

Granted that such income processes exist, one might still ask: are they at all empirically plausible? We shall present two specifications that match the broad facts of the U. S. income distribution and intergenerational persistence reasonably well. First, let $p = .55$, $q = .6$, $r = .5$, and $s = .7$, leading to the transition matrix,

$M = [\text{Formula Omitted}]$

and the stationary distribution $(\pi_1, \pi_2, \pi_3) = (.33, .44, .23)$. Thus, 77 percent of the population is always poorer than average, yet 67 percent always have expected income above average. In each period, however, only 23 percent actually end up with realized incomes above the mean, thus replicating the invariant distribution. Choosing $(a_1, a_2, a_3) = (16000, 36000, 91000)$, we obtain a rather good fit with the data, especially in light of the model's extreme simplicity; see Table I, columns 1 and 2.

This income process also has more persistence for the lower and upper income groups than for the middle class, which is consistent with the findings of Cooper, Durlauf, and Johnson [1994]. But most striking is its main political implication: a two-thirds majority of voters will support a policy or constitution designed to implement a zero tax rate for all future generations, even though:

—no deadweight loss concern enters into voters' calculations;

—three-quarters of the population is always poorer than average;

—the pivotal middle class, which accounts for most of the laissez-faire coalition, knows that its children have less than a one in three chance of "making it" into the upper class.

The last column of Table I presents the results for a slightly different specification, which also does a good job of matching the same key features of the data, and which we shall use later on when studying the effects of risk aversion. With $(p, q, r, s) = (.45, .6, .3, .7)$, and $(a_1, a_2, a_3) = (20000, 35000, 90000)$, the transition matrix is now

$M = [\text{Formula Omitted}]$

and the invariant distribution is $(\pi_1, \pi_2, \pi_3) = (.39, .37, .24)$. The majority opposing future redistributions is now a bit lower, but still 61 percent. Middle-class children now have about a 40 percent chance of upward mobility, and this will make a difference when we introduce risk aversion later on. Note that the source of these greater expected income gains is increased concavity in the transition process, relative to the first specification.

III.C. MULTIPERIOD REDISTRIBUTIONS

We now extend the general stochastic analysis to multiperiod redistributions, maintaining the assumption of risk neutrality (or complete markets). Agents thus care about the expected present value of their net income over the $T + 1$ periods during which the chosen tax scheme is to remain in place. For any individual i , we denote by $\Theta_i \equiv (\Theta_1^i, \dots, \Theta_t^i)$ the random sequence of shocks which she receives up to date t , and by $\theta_i \equiv (\theta_1^i, \dots, \theta_t^i)$ a sample realization. Given a one-step transition function $f \in T_p$, her income in period t is

$$(8) y_t^i = f(\dots, f(f(y_{t-1}^i; \theta_{t-1}^i); \theta_{t-2}^i); \dots; \theta_t^i) \equiv f^t(y_0^i; \theta_t^i), t = 1, \dots, T,$$

where $f^t(y_0^i; \theta_t^i)$ now denotes the t -step transition function. Under laissez-faire, the expected present value of this income stream over the political horizon is

$$V^T(y_0^i) \equiv E_{\theta_0^i} \dots E_{\theta_t^i} [\text{[Graphic Character Omitted]} \delta^t y_t^i | y_0^i] = \text{[Graphic Character Omitted]} \delta^t E_{\theta_0^i} f^t(y_0^i; \theta_t^i) = \text{[Graphic Character Omitted]} \delta^t E_{\theta_0^i} f^t(y_0^i; \theta_t^i),$$

where we suppressed the index i on the random variables θ_t^i since they all have the same probability distribution $P^t(\theta_t) \equiv \prod_{k=1}^t P(\theta_k)$ on Ω^t . Under the policy r_1 , on the other hand, agent i 's expected income at each t is the expected mean $E_{G_{J_t}}[\mu_{F_t}]$, which by Fubini's theorem is also the mean expected income. The resulting payoff is

$$\text{[Graphic Character Omitted]} \delta^t E[\mu_{F_t}] = \text{[Graphic Character Omitted]} \delta^t \left(\int_0^1 E[\text{[Graphic Character Omitted]}]_t [f^t(y_0^i; \theta_t^i)] dj \right) = \text{[Graphic Character Omitted]} \delta^t \int_X E_{\theta_t} [f^t(y; \theta_t)] dF_0(y),$$

so that agent i votes for r_1 over r_0 if and only if

$$(9) V^T(y_0^i) > \int_X V^T(y) dF_0(y).$$

It is easily verified that for transition functions which are concave (but not affine) in y with probability one, that is, for $f \in T_P$, every function $f^t(y; \theta_t)$, $t \geq 1$, inherits this property. Naturally, so do the weighted average $\sum_{t=0}^T \delta^t f^t(y; \theta_t)$ and its expectation $V^T(y)$, for $T \geq 1$. Hence, in this quite general setup, the now familiar result:

PROPOSITION 3. Let $F_0 \in F$, $\delta \in (0, 1]$, $T \geq 1$. For any mobility process (f, P) with $f \in T_P$, there exists a unique $y_f^*(T) < \mu_{F_0}$ such that all agents in $[0, y_f^*(T))$ vote for r_1 over r_0 , while all those in $(y_f^*(T), 1]$ vote for r_0 over r_1 .

Note that Proposition 3 does not cover the larger class of transition processes T_P^* defined earlier, since $V^T(y)$ need not be concave if f is only concave in expectation. Can one obtain a stronger result, similar to that of the deterministic case, namely that the tipping point decreases as the horizon lengthens? While this seems quite intuitive, and will indeed occur in the data analyzed in Section V, it may in fact not hold without relatively strong additional assumptions. The reason is that the expectation operator does not, in general, preserve the "more concave than" relation. A sufficient condition which insures this result is that the $t+1$ -step transition function be more concave in expectation than the t -step transition function.

PROPOSITION 4. Let $F_0 \in F$, $\delta \in (0, 1]$, $T \geq 1$, and let (f, P) be a mobility process with $f \in T_P^*$. If, for all t , $f^{t+1}(\cdot; \theta_{t+1})$ [Graphic Character Omitted] $_{P^{t+1}}$ $f^t(\cdot; \theta_t)$, that is, $E_{\theta_0} \dots E_{\theta_{t+1}} [f^{t+1}(M.; \theta_{t+1})]$ [Graphic Character Omitted] $E_{\theta_0} \dots E_{\theta_t} [f^t(M.; \theta_1, \dots, \theta_t)]$,

then the larger the political horizon T , the larger the share of the votes that go to r_0 .

The interpretation is the same as that of Theorem 2(a): the more forward-looking the voters, or the more long-lived the tax scheme, the lower is the political support for redistribution. An immediate corollary is that this monotonicity holds when the transition function is of the form $f(y, \theta) = y^\alpha \phi(\theta)$, where $\alpha \in (0, 1)$ and ϕ can be an arbitrary function. This is the familiar log linear model of income mobility, $\ln y_{t+1}^i = \alpha \ln y_t^i + \ln \epsilon_{t+1}^i$, which is widely used in the empirical literature.

IV. EXTENDING THE BASIC FRAMEWORK

IV.A. RISK-AVERSION

When agents are risk averse, the fact that redistributive policies provide insurance against idiosyncratic shocks increases the breadth of their political support. Consequently, the cutoff separating those who vote for r_0 from those who prefer r_1 may be above or below the mean, depending on the relative strength of the prospect of mobility and the risk-aversion effects. While the tension between these two forces is very intuitive, no general characterization of the cutoff in terms of the relative concavity of the transition and utility functions can be provided. To understand why, consider

again the simplest setup where agents vote at date 0 over the tax scheme for date 1. Denoting by U their utility function, the cutoff falls below the mean if

$$E_{\Theta}[U(f(E_{F_0}[y]; \Theta))] > U(E_{\Theta}E_{F_0}[f(y; \Theta)]),$$

where $E_{F_0}[y] = \mu_{F_0}$ denotes the expectation with respect to the initial distribution F_0 . Observe that $f(\cdot, \theta) \ni T$ if and only if the left-hand side is greater (for all U and P) than $E_{\Theta}[U(E_{F_0}[f(y; \Theta)])]$. But the concavity of U , namely risk aversion, is equivalent to the fact that this latter expression is also smaller than the right-hand side of the above inequality. The curvatures of the transition and utility functions clearly work in opposite directions, but the cutoff is not determined by any simple composite of the two.(FN18)

One can, on the other hand, assess the outcome of this “battle of the curvatures” quantitatively. To that effect, let us return to the simulations of the Markovian model reported in Table I, and ask the following question: how risk averse can the agents with median income a_2 be, and still vote against redistribution of future incomes based on the prospects of upward mobility? Assuming CRRA preferences and comparing expected utilities under r_0 and r_1 , we find that the maximum degree of risk aversion is only 0.35 under the specification of column 2, but rises to 1 under that of column 3. The second number is well within the range of plausible estimates, albeit still somewhat on the low side.(FN19) While the results from such a simple model need to be interpreted with caution, these numbers do suggest that, with empirically plausible income processes, the POUM mechanism will sustain only moderate degrees of risk aversion. The underlying intuition is simple: to offset risk aversion, the expected income gain from the POUM mechanism has to be larger, which means that the expected transition function must be more concave. In order to maintain a realistic invariant distribution, the skewed idiosyncratic shocks must then be more important, which is of course disliked by risk-averse voters.

IV.B. NONLINEAR TAXATION

Our analysis so far has mostly focused on an all-or-nothing policy decision, but we explained earlier that it immediately extends to the comparison of any two linear redistribution schemes. We also provided results on voting equilibrium within this class of linear policies. In practice, however, both taxes and transfers often depart from linearity. In this subsection we shall therefore extend the model to the comparison of arbitrary progressive and regressive schemes, and demonstrate that our main conclusions remain essentially unchanged.

When departing from linear taxation (and before mobility considerations are even introduced), one is inevitably confronted with the nonexistence of a voting equilibrium in a multidimensional policy space. Even the simplest cases are subject to this well-known problem.(FN20)

One can still, however, restrict voters to a binary policy choice (e.g., a new policy proposal versus the status quo), and derive results on pairwise contests between alternative nonlinear schemes. This is consistent with our earlier focus on pairwise contests between linear schemes, and will most clearly demonstrate the main new insights. In particular, we will show how Marhuenda and Ortuño-Ortín’s [1995] result for the static case may be reversed once mobility prospects are taken into account.

Recall that a general redistribution scheme was defined as a mapping $r: X \times F \rightarrow R_+$ which preserves total income. In what follows, we shall take the economy’s initial income distribution as given, and denote disposable income $r(y; F)$ as simply $r(y)$. A redistribution scheme r can then equivalently be specified by means of a tax function $T: X \rightarrow R$, with collected revenue $\int_X T(y) dF$ rebated lump-sum to all agents, and the normalization $T(0) \equiv 0$ imposed without loss of generality. Thus,

$$(10) \quad r(y) = y - T(y) + \int_X T(y) dF, \text{ for all } y \ni X.$$

We shall confine our attention to redistribution schemes with the following standard properties: T and r are increasing and continuous on X , with $\int_X T \, dF \geq 0$ and $r \geq 0$. We shall say that such a redistribution scheme is progressive (respectively, regressive) if its associated tax function is convex (respectively, concave). (FN21)

To analyze how nonlinear taxation interacts with the POUM mechanism, we maintain the simple two-period setup of subsections II.A and III.A, and consider voters faced with the choice between a progressive redistribution scheme and a regressive one (which could be laissez-faire). The key question is then whether political support for the progressive scheme is lower when the proposed policies are to be enacted next period, rather than in the current one. We shall show that the answer is positive, provided that the transition function f is concave enough relative to the curvature of the proposed (net) redistributive policy.

To be more precise, let r_{prog} and r_{reg} be any progressive and regressive redistribution schemes, with associated tax functions T_{prog} and T_{reg} . Let F_0 denote the initial income distribution, and y_{F_0} the income level where agents are indifferent between implementing r_{prog} or r_{reg} today. Given a mobility process $f \in T_P$, let y_f^* denote, as before, the income level where agents are indifferent between implementing r_{prog} or r_{reg} next period. The basic POUM hypothesis is that $y_f^* < y_{F_0}$, so that expectations of mobility reduce the political support for the more redistributive policy by $F_0(y_{F_0}) - F_0(y_f^*)$. We shall establish two propositions that provide sufficient conditions for an even stronger result, namely $y_{F_0} \geq \mu_{F_0} > y_f^*$. (FN22) In other words, whereas the coalition favoring progressivity in current fiscal policy extends to agents even richer than the mean [Marhuenda and Ortuño-Ortín 1995], the coalition opposing progressivity in future fiscal policy extends to agents even poorer than the mean.

The first proposition places no restrictions on F_0 but focuses on the case where the progressive scheme T_{prog} involves (weakly) higher tax rates than T_{reg} at every income level. In particular, it raises more total revenue.

PROPOSITION 5. Let $F_0 \in F$, let (f, P) be a mobility process with $f \in T_P^*$, and let $T_{\text{prog}}, T_{\text{reg}}$ be progressive and regressive tax schemes with $T'_{\text{prog}}(0) \geq T'_{\text{reg}}(0)$. If $E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(y, \Theta))]$ is concave (not affine), the political cutoffs for current and future redistributions are such that $y_{F_0} \geq \mu_{F_0} > y_f^*$.

Note that $T_{\text{prog}} - T_{\text{reg}}$ is a convex function, so the key requirement is that f be concave enough, on average, to dominate this curvature. The economic interpretation is straightforward: the differential in an agent's (or her child's) future expected tax bill must be concave in her current income.

Given the other assumptions, the boundary condition $T'_{\text{prog}}(0) \geq T'_{\text{reg}}(0)$ implies that the tax differential $T_{\text{prog}} - T_{\text{reg}}$ is always positive and increases with pretax income. This requirement is always satisfied when $T_{\text{reg}} \equiv 0$, which corresponds to the laissez-faire policy. On the other hand, it may be too strong if the regressive policy taxes the poor more heavily. It is dropped in the next proposition, which focuses on economies in steady-state and tax schemes that raise equal revenues.

PROPOSITION 6. Let (f, P) be a mobility process with $f \in T_P^*$, and denote by F the resulting steady-state income distribution. Let T_{prog} and T_{reg} be progressive and regressive tax schemes raising the same amount of steady-state revenue: $\int_X T_{\text{prog}} \, dF = \int_X T_{\text{reg}} \, dF$. If $E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(y, \Theta))]$ is concave (not affine) with $E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(\bar{y}, \Theta))] \geq 0$, then the political cutoffs for current and future redistributions are such that $y_F \geq \mu_F > y_f^*$.

The concavity requirement and its economic interpretation are the same as above. The boundary condition now simply states that agents close to the maximum income

\bar{y} face a higher expected future tax bill under T_{prog} than under T_{reg} . This requirement is both quite weak and very intuitive.

Propositions 5 and 6 demonstrate that the essence of our previous analysis remains intact when we allow for plausible nonlinear taxation policies. At most, progressivity may increase the extent of concavity in f required for the POUM mechanism to be effective.(FN23) Note also that all previous results comparing complete redistribution and laissez-faire ($T_{\text{prog}}(y) \equiv y$, $T_{\text{reg}}(y) \equiv 0$), or more generally two linear schemes with different marginal rates, immediately obtain as special cases.

V. MEASURING POUM IN THE DATA

The main objective of this paper was to determine whether the POUM hypothesis is theoretically sound, in spite of its apparently paradoxical nature. As we have seen, the answer is affirmative. The next question that naturally arises is whether the POUM effect is present in the actual data, and if so, whether it is large enough to matter for redistributive politics.

Our purpose here is not to carry out a large-scale empirical study or detailed calibration, but to show how the POUM effect can be measured quite simply from income mobility and inequality data. We shall again start with the benchmark case of risk-neutral agents, then introduce risk aversion. The first question that we ask is thus the following: at any given horizon, what is the proportion of agents who have expected future incomes strictly above the mean? In particular, does it increase with the length of the forecast horizon, and does it eventually rise above 50 percent?(FN24)

Rather than impose a specific functional form on the income process, we shall use here the more flexible description provided by empirical mobility matrices. These are often estimated for transitions between income quintiles, which is too coarse a grid for our purposes, especially around the median. We shall therefore use the more disaggregated data compiled by Hungerford [1993] from the Panel Study on Income Dynamics (PSID), namely,

(a) interdecile mobility matrices for the periods 1969-1976 and 1979-1986, denoted M_{69}^{76} and M_{79}^{86} , respectively. Each of those is in fact computed in two different ways: using annual family incomes, and using five-year averages centered on the first and last years of the transition period, so as to reduce measurement error.

(b) mean income for each decile, in 1969 and 1979. We shall treat each decile as homogeneous, and denote the vectors of relative incomes as a_{69} and a_{79} . A "prime" will denote transposition.

Let us start by examining these two income distributions:

$$\begin{aligned} a'_{69} &= (.211 \quad .410 \quad .566 \quad .696 \quad .822 \quad .947 \\ &\quad \quad \quad \quad \quad \quad \quad \quad 1.104 \quad 1.302 \quad 1.549 \quad 2.393), \\ a'_{79} &= (.179 \quad .358 \quad .523 \quad .669 \quad .801 \quad .933 \\ &\quad \quad \quad \quad \quad \quad \quad \quad 1.084 \quad 1.289 \quad 1.588 \quad 2.576). \end{aligned}$$

In both years the median group earned approximately 80 percent of mean income, while those with the average level of resources were located somewhere between the sixtieth and seventieth percentiles. More precisely, by linear interpolation we can estimate the size of the redistributive coalition to be 63.4 percent in 1969 and 64.4 percent in 1979.

Next, we apply the appropriate empirical transition matrix to compute the vector of conditionally expected relative incomes $t \times 7$ years ahead, namely $(M_{69}^{76})^t \cdot a_{69}$ or $(M_{79}^{86})^t \cdot a_{79}$, for $t = 1, \dots, 3$. The estimated rank of the cutoff y^*_t where expected future income equals the population mean is then obtained by linear interpolation of these decile values. By iterating a seven-year transition matrix to compute mobility over 14 and 21 years we are, once again, treating the transition process as stationary. Similarly, by using the initial income distribution vectors we are abstracting from

changes in the deciles' relative incomes during the transition period. These are obviously simplifying approximations, imposed by the limitations of the data.(FN25)

The results, presented in Table IIa, are consistent across all specifications: the POUM effect appears to be a real feature of the process of socioeconomic mobility in the United States—even at relatively short horizons, but especially over longer ones.(FN26) It affects approximately 3.5 percent of the population over seven years, and 10 percent over fourteen years. This is far from negligible, especially since the differential rates of political participation according to socioeconomic class observed in the United States imply that the pivotal agent is almost surely located above the fifty-fifth percentile, and probably above the sixtieth.(FN27) Over a horizon of approximately twenty years mobility prospects offset the entire 13-15 percent point interval between mean and median incomes, so that if agents are risk-neutral a strict majority will oppose such long-run redistributions. Thus, in both 1969 and 1979, 64 percent of the population was poorer than average in terms of current income, and yet 51 percent could rationally see themselves as richer than average in terms of expected income two decades down the road.

Upward mobility prospects for the poor, however, represent only one of the forces that determine the equilibrium rate of redistribution—alongside with deadweight losses, the political system, and especially risk aversion, which was seen to work in the opposite direction. The second question we consider is therefore how the POUM effect compares in magnitude with the demand for social insurance. To that end, let us now assume that agents have constant-relative risk aversion $\beta > 0$. Using the same procedure and horizons of $t \times 7$ years as before, we now compare each decile's expected utility under laissez-faire with the utility of receiving the mean income for sure. The threshold where they coincide is again computed by linear interpolation. The results, presented in Tables IIb to IIc, indicate that even small amounts of risk aversion will dominate upward mobility prospects in the expected utility calculations of the middle class. Beyond values of about $\beta = 0.25$ the threshold ceases to decrease with T , becoming instead either U-shaped or even monotonically increasing.

Our findings can thus be summarized as follows:

(a) a sizable fraction of the middle class can rationally look forward to expected incomes that rise above average over a horizon of ten to twenty years;

(b) on the other hand, these expected income gains are small enough, compared with the risks of downward mobility or stagnation, that they are not likely to have a significant impact on the political outcome, unless voters have very low risk aversion and discount rates.

Being drawn from such a limited empirical exercise, these conclusions should of course be taken with caution. A first concern might be measurement error in the PSID data underlying Hungerford's [1979] mobility tables. The use of family income rather than individual earnings, and the fact that replacing yearly incomes with five-year averages does not affect our results, suggest that this is probably not a major problem. A second caveat is that the data obviously pertain to a particular country, namely the United States, and a particular period. One may note, on the other hand, that the two sample periods, 1969-1976 and 1979-1986, witnessed very different evolutions in the distribution of incomes (relatively stable inequality in the first, rising inequality in the second), yet lead to very similar results. Still, things might well be different elsewhere, such as in a developing economy on its transition path. Finally, there is our compounding the seven-year mobility matrices to obtain forecasts at longer horizons. This was dictated by the lack of sufficiently detailed data on longer transitions, but may well presume too much stationarity in families' income trajectories.

The above exercise should thus be taken as representing only a first pass at testing for the POUM effect in actual mobility data. More empirical work will hopefully follow,

using different or better data to further investigate the issue of how people's subjective and, especially, objective income prospects relate to their attitudes vis-à-vis redistribution. Ravallion and Lokshin [2000] and Graham and Pettinato [1999] are recent examples based on survey data from post-transition Russia and post-reform Latin American countries, respectively. Both studies find a significant correlation between (self-assessed) mobility prospects and attitudes toward laissez-faire versus redistribution.

VI. CONCLUSION

In spite of its apparently overoptimistic flavor, the prospect of upward mobility hypothesis often encountered in discussions of the political economy of redistribution is perfectly compatible with rational expectations, and fundamentally linked to an intuitive feature of the income mobility process, namely concavity. Voters poorer than average may nonetheless opt for a low tax rate if the policy choice bears sufficiently on future income, and if the latter's expectation is a concave function of current income. The political coalition in favor of redistribution is smaller, the more concave the expected transition function, the longer the duration of the proposed tax scheme, and the more farsighted the voters. The POUM mechanism, however, is subject to several important limitations. First, there must be sufficient inertia or commitment power in the choice of fiscal policy, governing parties or institutions. Second, the other potential sources of curvature in voters' problem, namely risk aversion and nonlinearities in the tax system, must not be too large compared with the concavity of the transition function.

With the theoretical puzzle resolved, the issue now becomes an empirical one, namely whether the POUM effect is large enough to significantly affect the political equilibrium. We made a first pass at this question and found that this effect is indeed present in the U. S. mobility data, but likely to be dominated by the value of redistribution as social insurance, unless voters have very low degrees of risk aversion. Due to the limitations inherent in this simple exercise, however, only more detailed empirical work on mobility prospects (in the United States and other countries) will provide a definite answer.

ADDED MATERIAL

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TABLE I DISTRIBUTION AND PERSISTENCE OF INCOME IN THE UNITED STATES

	Data (1990)	Model 1	Model 2
Median family income (\$)	35,353	36,000	35,000
Mean family income (\$)	42,652	41,872	42,260
Standard deviation of family incomes (\$)	29,203	28,138	27,499
Share of bottom 100 * π_1 = 33% of population (%)	11.62	12.82	--

Share of middle 100 * π_2 = 44% of population (%)	39.36	37.46	--
Share of top 100 * π_3 = 24% of population (%)	49.02	49.72	--
Share of bottom 100 * π_1 % = 39% of population (%)	14.86	--	18.5
Share of middle 100 * π_2 % = 37% of population (%)	36.12	--	30.8
Share of top 100 * π_3 % = 24% of population (%)	49.02	--	50.8
Intergenerational correlation of log-incomes	0.35 to 0.55	.45	0.51

Sources: median and mean income are from the 1990 U. S. Census (Table F-5). The shares presented here are obtained by linear interpolation from the shares of the five quintiles (respectively, 4.6, 10.8, 16.6, 23.8, and 44.3 percent) given for 1990 by the U. S. Census Bureau (Income Inequality Table 1). The variance is computed from the average income levels of each quintile in 1990 (Table F-3). Estimates of the intergenerational correlation from PSID or NLS data are provided by Solon [1992], Zimmerman [1992], and Mulligan [1995], among others.

TABLE IIa INCOME PERCENTILE OF THE POLITICAL CUTOFF: RISK-NEUTRAL AGENTS

Forecast horizon (years)	0	7	14	21
1. Mobility matrix: M ⁷⁶⁶⁹				
Annual incomes	63.4	61.8	54.2	48.8
Five-year averages	63.4	60.8	56.4	52.9
2. Mobility matrix: M ⁸⁶⁷⁹				
Annual incomes	64.4	60.9	51.3	48.1
Five-year averages	64.4	58.8	54.3	51.4

TABLE IIb INCOME PERCENTILE OF THE POLITICAL CUTOFF: RISK AVERSION = 0.10

Horizon (years)	0	7	14	21
1. Mobility matrix: M ⁷⁶⁶⁹				
Annual incomes	63.4	62.5	60.8	60.2
Five-year averages	63.4	61.4	58.5	56.5
2. Mobility matrix: M ⁸⁶⁷⁹				
Annual incomes	64.4	61.7	56.6	53.0
Five-year averages	64.4	59.4	55.0	53.6

TABLE IIc INCOME PERCENTILE OF THE POLITICAL CUTOFF: RISK AVERSION = 0.25

Horizon (years)	0	7	14	21
1. Mobility matrix: M ⁷⁶⁶⁹				
Annual incomes	63.4	63.4	63.2	65.0
Five-year averages	63.4	62.4	61.3	61.5
2. Mobility matrix: M ⁸⁶⁷⁹				
Annual incomes	64.4	63.0	62.0	64.4
Five-year averages	64.4	60.3	57.6	56.9

TABLE II d INCOME PERCENTILE OF THE POLITICAL CUTOFF: RISK AVERSION = 0.50

Horizon (years)	0	7	14	21
1. Mobility matrix: M ⁷⁶⁶⁹				
Annual incomes	63.4	65.1	67.3	81.3
Five-year averages	63.4	64.3	65.6	69.1
2. Mobility matrix: M ⁸⁶⁷⁹				
Annual incomes	64.4	65.5	67.9	73.3
Five-year averages	64.4	62.0	61.3	64.3

FIGURE I Concave Transition Function

FIGURE II Expected Future Income under a Concave Transition Function (Semi-Logarithmic Scale)

FOOTNOTES

1. See, for example, Roemer [1998] or Putterman [1996]. The prospect of upward mobility hypothesis is also related to the famous “tunnel effect” of Hirschman [1973], although the argument there is more about how people make inferences about their mobility prospects from observing the experience of others. There are of course several other explanations for the broader question of why the poor do not expropriate the rich. These include the deadweight loss from taxation (e.g., Meltzer and Richard [1981]), and the idea that the political system is biased against the poor [Peltzman 1980; Bénabou 2000]. Putterman, Roemer, and Sylvestre [1999] provide a review.

2. If agents have a tendency toward overoptimistic expectations (as suggested by a certain strand of research in social psychology), this will of course reinforce the mechanism analyzed in this paper, which operates even under full rationality. It might be interesting, in future research, to extend our analysis of mobility prospects to a setting where agents have (endogenously) self-serving assessments of their own abilities, as in Bénabou and Tirole [2000].

3. For instance, Putterman, Roemer, and Sylvestre [1999] state that “voting against wealth taxation to preserve the good fortune of one’s family in the future cannot be part of a rational expectations equilibrium, unless the deadweight loss from taxation is expected to be large or voters are risk loving over some range.”

4. The concavity of f implies that $f(x)/x$ is decreasing, which corresponds to “tax” progressivity and Lorenz equalization, on any interval $[y, \bar{y}]$ such that $yf'(y) \leq f(y)$. This clearly applies in the present case, where $y = 0$ and $f(0) \geq 0$ since income is nonnegative. When the boundary condition does not hold, concavity is consistent with (local or global) regressivity. At the most general level, a concave scheme is thus one that redistributes from the extremes toward the mean. This is the economic meaning of Jensen’s inequality, given the normalization $\mu' = \mu$. In practice, however, most empirical mobility processes are clearly progressive (in expectation). The progressive case discussed above and illustrated in Figure I is thus really the relevant one for conveying the key intuition.

5. Another analytical example is the log linear, lognormal $ar(1)$ process commonly used in econometric studies. Complete closed-form solutions to the model under this specification are provided in Bénabou and Ok [1998].

6. More generally, the income support could be any interval $[y, \bar{y}]$, $y \geq 0$. We choose $y = 0$ for notational simplicity.

7. See, for instance, Meltzer and Richard [1981], Persson and Tabellini [1996], or Alesina and Rodrik [1994]. Proportional schemes reduce the voting problem to a single-dimensional one, thereby allowing the use of the median voter theorem. By contrast, when unrestricted nonlinear redistributive schemes are allowed, there is generally no voting equilibrium (in pure strategies): the core of the associated voting game is empty.

8. Another possible channel through which current tax decisions might incorporate concerns about future redistributions is if voters try to influence future political outcomes by affecting the evolution of the income distribution, through the current tax rate. This strategic voting idea has little to do with the POUM hypothesis as discussed in the literature (see references in footnote 1, as well as Okun’s citation). Moreover, these dynamic voting games are notoriously intractable, so the nearly universal practice in political economy models is to assume “myopic” voters. In our model the issue does

not arise since we focus on endowment economies, where income dynamics are exogenous.

9. We thus consider only endowment economies, but the POUM mechanism remains operative when agents make effort and investment decisions, and the transition function varies endogenously with the chosen redistributive policies. Bénabou [1999, 2000] develops such a model, using specific functional and distributional assumptions.

10. Throughout the paper we shall follow the common practice of ignoring the subtle mathematical problems involved with continua of independent random variables, and thus treat each Θ_i^t as jointly measurable, for any t . Consequently, the law of large numbers and Fubini's theorem are applied as usual.

11. For any r_τ and $r_{\tau'}$ in P such that $0 \leq \tau < \tau' \leq 1$, agent $y \leq X$ votes for r_τ over $r_{\tau'}$ iff $(1 - \tau) f(y) + \tau \mu_{F_1} < (1 - \tau') f(y) + \tau' \mu_{F_1}$, which in turn holds if and only if (2) holds. Thus, as noted earlier, nothing is lost by focusing only on the two extreme schemes in P , namely r_0 and r_1 .

12. In this case, r_0 is the unique Condorcet winner in P . Note also that Theorem 1—like every other result in the paper concerning median income m_{F_0} —holds in fact for any arbitrary income cutoff below μ_{F_0} (see the proof in the Appendix).

13. The reason why δ and T must be large enough in part (b) of Theorem 2 is that redistribution is now assumed to be implemented right away, starting in period 0. If it takes effect only in period 1, as in the previous section, the results apply for all δ and $T \geq 1$.

14. Interestingly, $>$ and $>_p$ are logically independent orderings. Even if there exists some $h \leq T$ such that $f(\cdot, \theta) = h(g(\cdot, \theta))$ for all θ , it need not be that $f >_p g$.

15. The simplest case where the distribution of expectations and the distribution of realizations differ is that where future income is the outcome of a winner-take-all lottery. The first distribution reduces to a single mass-point (everyone has the same expected payoff), whereas the second is extremely unequal (there is only one winner). Note, however, that this income process does not have the POUM property (instead, everyone's expected income coincides with the mean), precisely because it is nowhere strictly concave.

16. If the inequality $y_f^* < \mu_F$ is required to hold only for the steady-state distribution(s) F induced by f and P , rather than for all initial distributions, concavity of $E_\Theta[f(\cdot; \Theta)]$ is still a sufficient condition, but no longer a necessary one. Nonetheless, some form of concavity "on average" is still required, so to speak: if $E_\Theta[f(\cdot; \Theta)]$ were linear or convex, we would have $y_f^* \geq \mu_F$ for all distributions, including stationary ones.

17. Put differently, we posit that $M = [m_{kl}]_{3 \times 3}$ is a monotone transition matrix, requiring row $k + 1$ to stochastically dominate row k : $m_{11} \geq m_{21} \geq m_{31}$ and $m_{11} + m_{12} \geq m_{21} + m_{22} \geq m_{31} + m_{32}$. Monotone Markov chains were introduced by Keilson and Ketser [1977], and applied to the analysis of income mobility by Conlisk [1990] and Dardanoni [1993].

18. One case where complete closed-form results can be obtained is for the log linear specification with lognormal shocks; see Bénabou and Ok [1998].

19. Conventional macroeconomic estimates and values used in calibrated models range from .5 to 4, but most cluster between .5 and 2. In a recent detailed study of the income and consumption profiles of different education and occupation groups, Gourinchas and Parker [1999] estimate risk aversion to lie between 0.5 and 1.0.

20. For instance, if the policy space is that of piecewise linear, balanced schemes with just two tax brackets (which has dimension three), there is never a Condorcet winner. Even if one restricts the dimensionality further by imposing a zero tax rate for the first bracket, which then corresponds to an exemption, the problem remains: there exist many (positively skewed) distributions for which there is no equilibrium.

21. Recall that with $T(0) = 0$, if T is convex (marginally progressive), then it is also progressive in the average sense; i.e., $T(x)/x$ is increasing. Similarly, if T is concave, it is regressive both in the marginal and in the average sense.
22. The first inequality is strict unless $T_{\text{reg}} - T_{\text{prog}}$ happens to be linear. The above discussion implicitly assumes that voter preferences satisfy a single-crossing condition, so that y_{F_0} and y_f^* exist and are indeed tipping points. Such will be the case in our formal analysis.
23. Recall that Propositions 5 and 6 provide sufficient conditions for $y_f^* < \mu_{F_0} \leq y_{F_0}$. They are thus probably stronger than actually necessary, especially for the basic POUM result, which is only that $y_f^* < y_{F_0}$.
24. Recall that there is no reason a priori (i.e., absent some concavity in the transition function) why either effect should be observed in the data, since these are not general features of stationary processes. Rather than just “mean-reverting,” the expected income dynamics must be “mean-crossing from below” over some range, as in Figure II.
25. As a basic robustness check, we used the composite matrix $M_{69}^{76} \cdot M_{79}^{86}$ to recompute the fourteen-year transitions. We also applied the transition matrices M_{69}^{76} , $M_{69}^{76} \cdot M_{79}^{86}$ and relevant iterations to the (slightly more unequal) income distribution a_{79} , instead of a_{69} . The results (not reported here) did not change.
26. Tracing the effect back to its source, one can also examine to what extent expected future income is concave in current income. The expected transition is in fact concave over most, but not all, of its domain; of the nine slopes defined by the ten decile values, only three are larger than their predecessor when we use $(M_{69}^{76}; a_{69})$, and only two when we use $(M_{79}^{86}; a_{79})$. Recall that while concavity at every point is always a sufficient condition for the POUM effect, it is a necessary one only if one requires $y_f^* < \mu_{F_0}$ to hold for any initial F_0 . For a given initial distribution, such as the one observed in the data, there must be simply “enough” concavity on average, so that Jensen’s inequality is satisfied. This is the situation encountered here.
27. Using data on how the main forms of political participation (voting, trying to influence others, contributing money, participating in meetings and campaigns, etc.) vary with income and education, Bénabou [2000] computes the resulting bias with respect to the median. It is found to vary between 6 percent for voting propensities and 24 percent for propensities to contribute money, with most values being above 10 percent.

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APPENDIX

PROOF OF PROPOSITION 2

By using Jensen's inequality, we observe that f [Graphic Character Omitted] g implies that

$$f(y_f^*) = \int_X f dF_0 = \int_X h(g) dF_0 < h(\int_X g dF_0) = h(g(y_g^*)) = f(y_g)$$

for some $h \in T$. The proposition follows from the fact that f is strictly increasing.

QED

PROOF OF THEOREM 1

Let $F_0 \ni F_+$, so that the median m_{F_0} is below the mean μ_{F_0} . More generally, we shall be interested in any income cutoff $\eta < \mu_{F_0}$. Therefore, define for any $\alpha \in [0,1]$ and any $\eta \in (0, \mu_{F_0})$, the function

$$(A.1) \quad g_{\eta, \alpha}(y) \equiv \min [y, \eta + \alpha (y - \eta)],$$

which clearly is an element of T . It is clear that

$$\int_X g_{\eta, 0} dF_0 < \eta < \mu_{F_0} = \int_X g_{\eta, 1} dF_0,$$

So by continuity there exists a unique $\alpha(\eta) \in (0,1)$ which solves

$$(A.2) \quad \int_X g_{\eta, \alpha(\eta)} dF_0 = \eta.$$

Finally, let $f = g_{\eta, \alpha(\eta)}$, so that $\mu_{F_1} = \eta$ by (A.2). Adding the constant $\mu_{F_0} - \eta$ to the function f so as to normalize $\mu_{F_1} = \mu_{F_0}$ would of course not alter any of what follows. It is clear from (A.1) that everyone with $y < \eta$ prefers r_0 to r_1 , while the reverse is true for everyone with $y > \eta$; so $y_f^* = \eta$. By Proposition 2, therefore, the fraction of agents who support redistribution is greater (respectively, smaller) than $F(\eta)$ for all $f \in T$ which are more (respectively, less) concave than f . In particular, choosing $\eta = m_{F_0} < \mu_{F_0}$ yields the claimed results for majority voting. QED

PROOF OF THEOREM 2

For each $t = 1, \dots, T$, $f^t \in T$, so by Proposition 1 there is a unique $y_f^{*t} \in (0, \mu_{F_t})$ such that $f^t(y_f^{*t}) = \mu_{F_t}$. Moreover, since f^T [Graphic Character Omitted] f^{T-1} [Graphic Character Omitted] ... [Graphic Character Omitted] f , Proposition 2 implies that $y_f^{*T} < y_f^{*T-1} < \dots < y_f^{*1} < y_f^0 \equiv \mu_{F_0}$. The concavity of f also implies $f(\mu_{F_t}) > \mu_{F_{t+1}}$ for all t , from which it follows by a simple induction that

$$(A.3) \quad f^t(\mu_{F_0}) \geq \mu_{F_t} = f^t(y_f^{*t})$$

with strict inequality for $t > 1$ and $t < T$, respectively. Let us now define the operators $V^T : T \rightarrow T$ and $W^T : T \rightarrow R$ as follows:

$$(A.4) \quad V^T(f) \equiv [\text{Graphic Character Omitted}] \delta^t f^t \text{ and}$$

$$W^T(f) \equiv \int_X V^T(f) dF_0 = [\text{Graphic Character Omitted}] \delta^t \mu_{F_t}.$$

Agent y achieves utility $V^T(f)(y)$ under laissez-faire, and utility $W^T(f)$ under the redistributive policy. Moreover, (A.3) implies that

$$V^T(f)(\mu_{F_0}) = [\text{Graphic Character Omitted}] \delta^t f^t(\mu_{F_0}) > [\text{Graphic Character Omitted}] \delta^t \mu_{F_t}$$

$$> [\text{Graphic Character Omitted}] \delta^t f^t(y_f^{*t}) = V^T(f)(y_f^{*t})$$

for any $T \geq 1$. Since $V^T(f)(\cdot)$ is clearly continuous and increasing, there must therefore exist a unique $y_f^*(T) \in (y_f^{*T}, \mu_{F_0})$ such that

$$(A.5) \quad V^T(f)(y_f^*(T)) = [\text{Graphic Character Omitted}] \delta^t \mu_{F_t} = W^T(f).$$

But since $y_f^{*T+1} < y_f^{*T}$, we have $y_f^{*T+1} < y_f^*(T)$. This implies that $\mu_{F_{T+1}} = f^{T+1}(y_f^{*T+1}) < f^{T+1}(y_f^*(T))$, and hence

$$V^{T+1}(f)(y_f^*(T)) = [\text{Graphic Character Omitted}] \delta^{t+1} f^{T+1}(y_f^*(T)) > [\text{Graphic Character Omitted}] \delta^{t+1} \mu_{F_{T+1}} = V^{T+1}(f)(y_f^*(T+1)).$$

Therefore, $y_f^*(T+1) < y_f^*(T)$ must hold. By induction, we conclude that $y_f^*(T') < y_f^*(T)$ whenever $T' > T$; part (a) of the theorem is proved.

To prove part (b), we shall use again the family of piecewise linear functions $g_{\eta, \alpha}$ defined in (A.1). Let us first observe that the t th iteration of $g_{\eta, \alpha}$ is simply

$$(A.6) \quad (g_{\eta, \alpha})^t(y) = \min [y, \eta + \alpha^t (y - \eta)] = g_{\eta, \alpha^t}(y).$$

In particular, both $g_{\eta,1} : y$ [Graphic Character Omitted] y and $g_{\eta,0} : y$ [Graphic Character Omitted] $\min [y, \eta]$ are idempotent. Therefore,

$$V^T(g_{\eta,1})(\eta) = [\text{Graphic Character Omitted}] \delta^t \eta < [\text{Graphic Character Omitted}] \delta^t \mu_{F_0} = W^T(g_{\eta,1}).$$

On the other hand, when the transition function is $g_{\eta,0}$, the voter with initial income η prefers r_0 (under which she receives η in each period) to r_1 , if and only if

$$\eta + [\text{Graphic Character Omitted}] \delta^t \eta = V^T(g_{\eta,0})(\eta) > W^T(g_{\eta,0}) = \mu_{F_0} + [\text{Graphic Character Omitted}] \delta^t \int_X \min [y, \eta] dF_0,$$

or, equivalently,

$$(A.7) (\mu_{F_0} - \eta) / (\eta - \int_X \min [y, \eta] dF_0) < [\text{Graphic Character Omitted}] \delta^t = \delta(1 - \delta^T) / (1 - \delta).$$

This last inequality is clearly satisfied for $(\delta, 1/T)$ close enough to $(1, 0)$. In that case, we have $W^T(g_{\eta,1}) > \sum_{t=0}^{T-1} \delta^t \eta > W^T(g_{\eta,0})$. Next, it is clear from (A.4) and (A.1) that $W^T(g_{\eta,\alpha})$ is continuous and strictly increasing in α . Therefore, there exists a unique $\alpha(\eta) \in (0, 1)$ such that $W^T(g_{\eta,\alpha(\eta)}) = \sum_{t=0}^{T-1} \delta^t \eta$. This means that, under the transition function $f \equiv g_{\eta,\alpha(\eta)}$, we have

$$W^T(f) = [\text{Graphic Character Omitted}] \delta^t f(\eta) = V^T(f)(\eta),$$

so that an agent with initial income η is just indifferent between receiving her laissez-faire income stream, equal to η in every period, and the stream of mean incomes μ_{F_1} . Moreover, under laissez-faire an agent with initial $y < \eta$ receives y in every period, while an agent with $y > \eta$ receives $\eta + \alpha(y - \eta) > \eta$. Therefore, η is the cutoff $y^*_f(T)$ separating those who support r_0 from those who support r_1 , given $f \equiv g_{\eta,\alpha(\eta)}$. This proves the first statement in part (b) of the theorem.

Finally, by part (a) of the theorem, increasing (decreasing) the horizon T will reduce (raise) the cutoff $y^*_f(T)$ below (above) η . Applying these results to the particular choice of a cutoff equal to median income, $\eta \equiv m_{F_0}$, completes the proof.

QED

PROOF OF THEOREM 3

As in the proof Theorem 1, let $F_0 \ni F_+$, and consider any income cutoff $\eta < \mu_{F_0}$. Recall the function $g_{\eta,\alpha(\eta)}(y)$ which was defined by (A.1) and (A.2) so as to ensure that $\mu_{F_1} = \eta$. (Once again, adding any positive constant to f would not change anything.) For brevity, we shall now denote $\alpha(\eta)$ and $g_{\eta,\alpha(\eta)}$ as just α and g . Let us now construct a stochastic transition function whose expectation is g and which, together with F_0 , results in a positively skewed F_1 . Let $p \in (0, 1)$, and let Θ be a random variable taking values 0 and 1 with probabilities p and $1 - p$. For any $\varepsilon \in (0, \eta)$, we define $f : X \times \Omega \rightarrow R^+$ as follows:

* if $0 \leq y \leq \eta - \varepsilon$, $f(y; \theta) \equiv y$ for all θ

* if $\eta - \varepsilon < y \leq \eta$,

$f(y; \theta) \equiv \eta - \varepsilon$ if $\theta = 0$ (probability p)

$f(y; \theta) \equiv (y - p(\eta - \varepsilon)) / (1 - p)$ if $\theta = 1$ (probability $1 - p$)

* if $\eta < y \leq \bar{y}$,

$f(y; \theta) \equiv \eta - \varepsilon$ if $\theta = 0$ (probability p)

$f(y; \theta) \equiv \eta + (\alpha(y - \eta) + p\varepsilon) / (1 - p)$ if $\theta = 1$ (probability $1 - p$).

By construction, $E_{\Theta}[f(y; \Theta)] = g(y)$ for all $y \in X$; therefore, $E_{\Theta}[f(\cdot; \Theta)] = g \in T$. It remains to be checked that $f(y; \Theta)$ is strictly stochastically increasing in y . In other words, for any $x \in X$, the conditional distribution $M(x|y) \equiv P([\theta \in \Omega | f(y; \theta) \leq x])$ must be decreasing in y on X , and strictly decreasing on a nonempty subinterval of X . But this is equivalent to saying that $\int_X h(x) dM(x|y)$ must be (strictly) increasing in y , for any (strictly) increasing function $h : X \rightarrow R$; this latter form of the property is easily verified from the above definition of $f(y; \theta)$.

Because $E_{\Theta}[f(\cdot; \Theta)] = g$, so that $\mu_{F_1} = \eta$ by (A.2), the cutoff between the agents who prefer r_0 and those who prefer r_1 is $y_f^* = \eta$. This tipping point can be set to any value below μ_{F_0} (i.e., $1 - F_0(\eta)$ can be made arbitrarily small), while simultaneously ensuring that $F_1(\mu_{F_1}) > \sigma$. Indeed,

$$F_1(\mu_{F_1}) = F_1(\eta) = p \int_X 1_{[f(x,0) \leq \eta]} dF_0(x) + (1 - p) \int_X 1_{[f(x,1) \leq \eta]} dF_0(x) = p + (1 - p)F_0(\eta - p\vartheta),$$

so by choosing p close to 1 this expression can be made arbitrarily close to 1, for any given η .

To conclude the proof, it only remains to observe that a transition function $f^* \ni T$ is more concave than f in expectation if and only if $E_{\Theta}[f^*(\cdot; \Theta)]$ [Graphic Character Omitted] $E_{\Theta}[f(\cdot; \Theta)] = g$. Proposition 2 then implies that the fraction of agents who support redistribution under f^* is greater than $F_0(\eta)$. The reverse inequalities hold whenever f [Graphic Character Omitted]_p f^* . As before, choosing the particular cutoff $\eta = m_{F_0} < \mu_{F_0}$ yields the claimed results pertaining to majority voting, for any distribution $F_0 \ni F_+$.

QED

PROOF OF PROPOSITION 4

Define $h_t = E_{\Theta_1} \dots E_{\Theta_t} f(\cdot; \Theta_1, \dots, \Theta_t)$, $t = 1, \dots$, and observe that $h_t \ni T$ and h_{t+1} [Graphic Character Omitted] h_t for all t under the hypotheses of the proposition. The proof is thus identical to part (a) of Theorem 2, with h_t playing the role of f^t .||

PROOF OF PROPOSITION 5

1) Properties of y_{F_0} . By the mean value theorem, there exists a point $y_{F_0} \ni X$ such that

$$(A.8) \quad (T_{\text{prog}} - T_{\text{reg}})(y_{F_0}) = \int_X (T_{\text{prog}} - T_{\text{reg}}) dF_0,$$

which by (10) means that $r_{\text{prog}}(y_{F_0}) = r_{\text{reg}}(y_{F_0})$. Now, since $T_{\text{prog}} - T_{\text{reg}}$ is convex, Jensen's inequality allows us to write

$$(A.9) \quad (T_{\text{prog}} - T_{\text{reg}})(\mu_{F_0}) \leq \int_X (T_{\text{prog}} - T_{\text{reg}}) dF_0 = (T_{\text{prog}} - T_{\text{reg}})(y_{F_0}).$$

With $(T_{\text{prog}} - T_{\text{reg}})'(0) \geq 0$, the convexity of $T_{\text{prog}} - T_{\text{reg}}$ implies that it is a nondecreasing function on $[0, \bar{y}]$. In the trivial case where it is identically zero, indifference obtains at every point, so choosing $y_F = \mu_{F_0}$ immediately yields the results. Otherwise, two cases are possible. Either $T_{\text{prog}} - T_{\text{reg}}$ is a strictly increasing linear function; or else it is convex (not affine) on some subinterval, implying that the inequality in (A.9) is strict. Under either scenario (A.8) implies that y_{F_0} is the unique cutoff point between the supporters of the two (static) policies, and (A.9) requires that $y_{F_0} \geq \mu_{F_0}$.

2) Properties of y_f^* . By the mean value theorem, there exists a point $y_f^* \ni X$ such that

$$(A.10) \quad E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(y_f^*, \Theta))] = \int_X E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(y, \Theta))] dF_0(y),$$

or equivalently $E_{\Theta}[r_{\text{prog}}(f(y_f^*, \Theta))] = E_{\Theta}[r_{\text{reg}}(f(y_f^*, \Theta))]$. Moreover,

$$(A.11) \quad \int_X E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(y, \Theta))] dF_0(y) < E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(\mu_{F_0}, \Theta))],$$

by Jensen's inequality for the concave (not affine) function $E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}}) f(\cdot, \Theta)]$ [Graphic Character Omitted] $f(\cdot, \Theta)$. This function is also nondecreasing, since $T_{\text{prog}} - T_{\text{reg}}$ has this property and $f \ni TP$ is increasing in y with probability one. Therefore, y_f^* is indeed the cutoff point such that $E_{\Theta}[r_{\text{prog}}(f(y, \Theta))] = E_{\Theta}[r_{\text{reg}}(f(y, \Theta))]$ as y [Graphic Character Omitted] y_f^* , and moreover (A.11) requires that $\mu_{F_0} > y_f^*$.

QED

PROOF OF PROPOSITION 6

1) Properties of y_F . The proofs of (A.8) and (A.9) proceed as above, except that the right-hand side of (A.8) is now equal to zero due to the equal-revenue constraint. Since

$T_{\text{prog}} - T_{\text{reg}}$ is convex, equal to zero at $y = 0$, and has a zero integral, it must again be the case (barring the trivial case where it is zero everywhere) that it is first negative on $(0, y_F)$, then positive on (y_F, \bar{y}) . Therefore, y_F is indeed the threshold such that $r_{\text{prog}}(y)$ [Graphic Character Omitted] $r_{\text{reg}}(y)$ as y [Graphic Character Omitted] y_F , and moreover, by (A.8) it must be that $\mu_F \leq y_F$.

2) Properties of y_f^* . Since the distribution F is, by definition, invariant under the mobility process (f, P) , so is the revenue raised by either tax scheme. This implies that the function $E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(y_f^*, \Theta))]$, like $T_{\text{prog}} - T_{\text{reg}}$, sums to zero over $[0, \bar{y}]$:

$$(A.12) \int_{\mathcal{X}} E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(y, \Theta))] dF(y) = \int_{\mathcal{X}} \int_{\Omega} (T_{\text{prog}} - T_{\text{reg}})(f(y, \theta)) dF(y) dP(\theta) = \int_{\mathcal{X}} (T_{\text{prog}} - T_{\text{reg}})(x) dF(x) = 0.$$

Therefore, by the mean value theorem there is at least one point y_f^* where

$$E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(y_f^*, \Theta))] = 0.$$

Furthermore, since this same function is concave (not affine), Jensen's inequality implies that

$$(A.13) E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(\mu_F, \Theta))] > \int_{\mathcal{X}} E_{\Theta}[(T_{\text{prog}} - T_{\text{reg}})(f(y, \Theta))] dF(y) = 0.$$

Finally, observe that since the function $E_{\Theta}[(r_{\text{prog}} - r_{\text{reg}})(f(\cdot, \Theta))]$ is convex (not affine) on $[0, \bar{y}]$, equal to zero at y_f^* and nonnegative at the right boundary \bar{y} , it must first be negative on $(0, y_f^*)$, then positive on (y_f^*, \bar{y}) . Therefore, y_f^* is indeed the unique tipping point, meaning that $E_{\Theta}[r_{\text{prog}}(f(y, \Theta))]$ [Graphic Character Omitted] $E_{\Theta}[r_{\text{reg}}(f(y, \Theta))]$ as y [Graphic Character Omitted] y_f^* ; moreover, by (A.13) we must have $\mu_F > y_f^*$.

QED